# IV. Principles of rotating machines

## Terms & Definitions

**Constructional viewpoint:** There are two mechanical parts of every rotating machine:

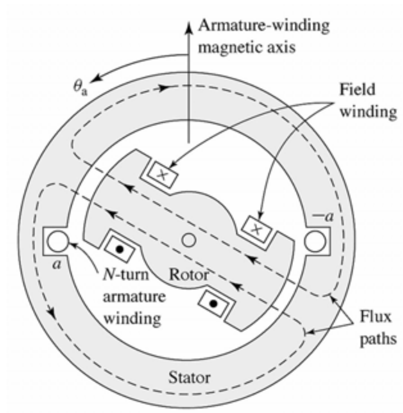
- **rotor:** inner rotating member
- **stator:** outer stationary member

**Operational viewpoint:** There are two main parts :

- **field:** incorporates field winding when excited field produces the main flux in the M.C. (primary source of flux)
- **armature:** incorporates the armature winding. This is the side at which the work is done. Armature react upon the field to produce motoring or generating torques

## Construction of Rotating Machines

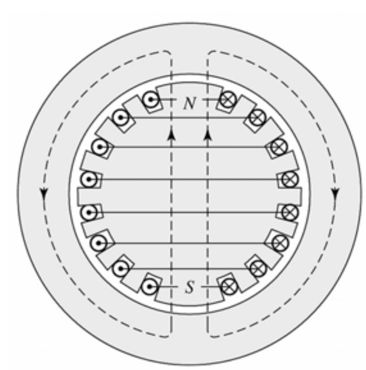
(a) Salient-pole structure in which the coil windings are concentrated around protruding poles




A simple, two-pole, single-phase salient-pole synchronous generator

## Construction of Rotating Machines

(b) Cylindrical structure in which the windings are distributed in slots cut into a cylindrical magnetic structure




Elementary two-pole cylindrical-rotor field winding.

## Types of Rotating Machines

The field & the armature sides can be placed on the stator or rotor sides depending on the machine type:

**(a) DC machines**
*(rotor is cylindrical, stator is salient-pole)*

- Field is on stator
- Armature is on rotor

**(b) Induction machines**
*(both stator and rotor are cylindrical)*

- Field is on stator
- Armature is on rotor

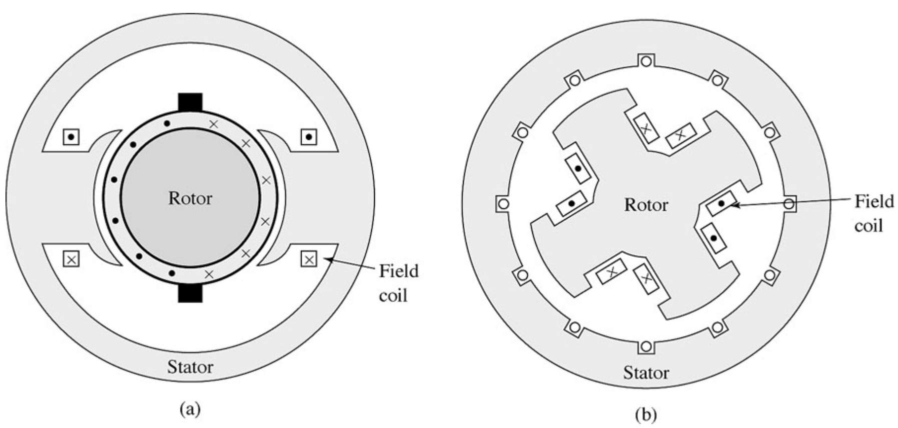
**(c) Synchronous machines**
*stator is cylindrical, rotor is either salient-pole or cylindrical)*

- Field is on rotor
- Armature is on stator

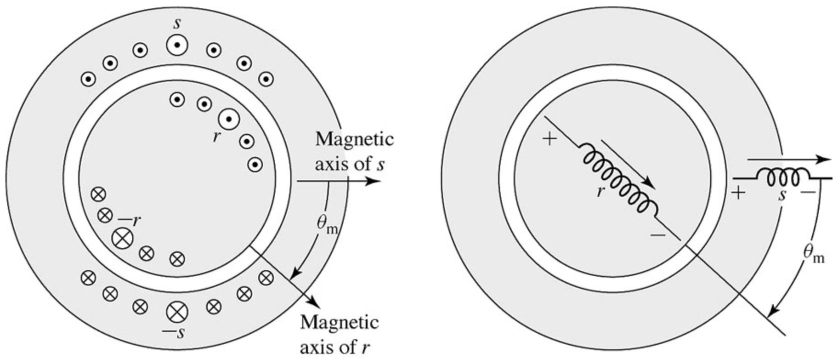
**Type of Windings:**

(a) Distributed type (e.g. armature winding of DC machines)
(b) Concentrated type (e.g. field winding of DC machines)

**Structure of typical salient-pole machines: (a) dc machine and (b) salient-pole synchronous machine.**



**Elementary two-pole machine with smooth air gap (cylindrical machine) : (a) winding distribution and (b) schematic representation.**



(a) (b)

## Voltage Generation in Rotating Machines

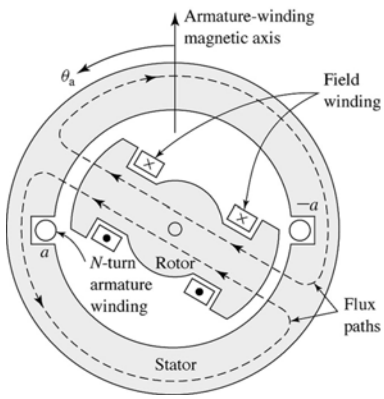
**Voltages are generated across armature windings:**

(i) by rotating armature winding mechanically through the magnetic field created by field winding

(ii) by rotating magnetic field past the armature winding

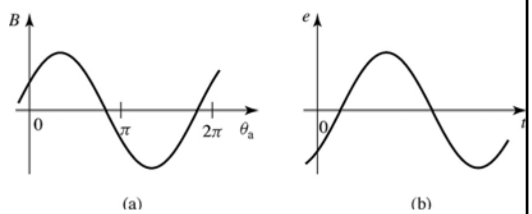
(iii) by the designing the magnetic circuit so that reluctance varies with the position of rotor

## Voltage Generation in Rotating Machines

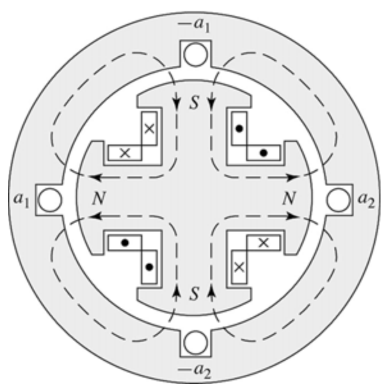



Schematic view
of a simple, two-pole, single-
phase synchronous generator.

**(a) Space distribution of flux density and**
**(b) corresponding waveform of the generated**
**voltage for the single-phase generator**

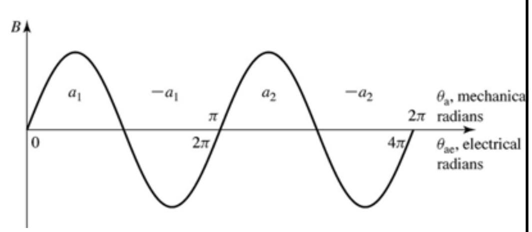



## Voltage Generation in Rotating Machines




Schematic view
of a simple, four-pole,
single-phase synchronous
generator

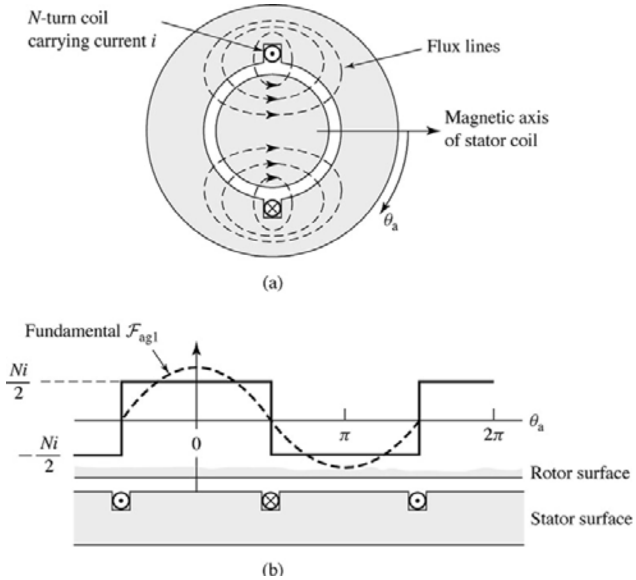
**Space distribution of the air-gap flux density**
**in a idealized, four-pole synchronous generator.**



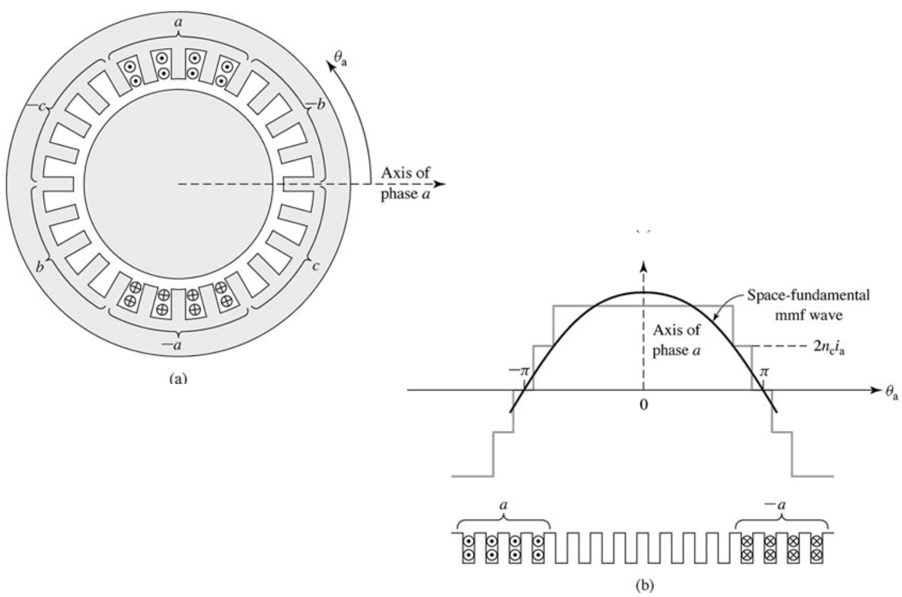
## MMF Waveforms in Rotating Machines

**(a) Schematic view of flux produced by a concentrated, full-pitch winding in a machine with a uniform air gap.**

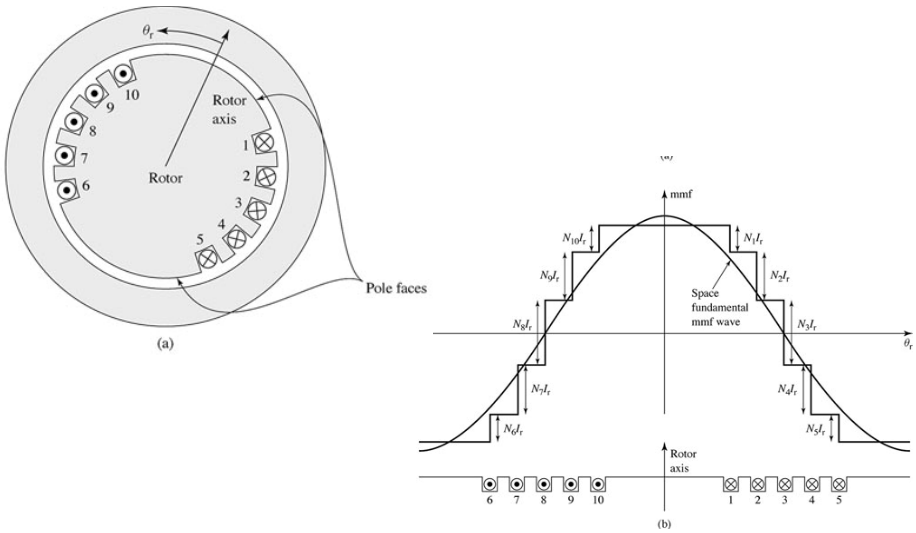
**(b) (b) The air-gap mmf produced by current in this winding.**




**The mmf of one phase of a distributed two-pole, three-phase winding with full-pitch coils.**

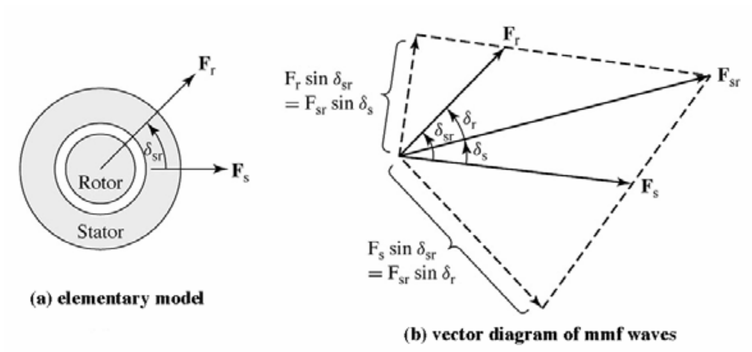


**The air-gap mmf of a distributed winding on the rotor of a round-rotor generator.**




## Torque Production in Rotating Machines

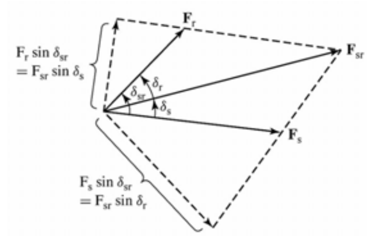
**MMF space vectors of a simplified two-pole machine**




**Torque is produced by the tendency of the rotor and stator magnetic fields to align.**

**Note that these figures are drawn with $\delta_{sr}$ positive, i.e., with the rotor mmf wave $F_r$ leading that of the stator $F_s$.**

## MMF Calculations:




$$\theta = \delta_{sr}$$
$$\mathscr{F}_a = F_r$$
$$\mathscr{F}_f = F_s$$

$$\overline{\mathscr{F}_{sr}}(\theta) = \overline{\mathscr{F}_f}(\theta) + \overline{\mathscr{F}_a}(\theta)$$

$$\mathscr{F}_{sr}^2 = \mathscr{F}_f^2 + \mathscr{F}_a^2 + 2\,\mathscr{F}_f\,\mathscr{F}_a \cos(\theta)$$

$$H_{sr} = \frac{\mathscr{F}_{sr}}{g}$$

Field coenergy density $\quad w'_f = \frac{1}{2}\mu_0 H_{sr}^2(\theta)$

Average field coenergy density $\quad w'_{fld}(\theta) = \frac{1}{2}\mu_0 \frac{H_{sr}^2}{2} = \frac{1}{4}\mu_0 \frac{\mathscr{F}_{sr}^2}{g^2}$

## Torque production:

$$W'_{fld} = w'_{fld} V_{gap} = \frac{1}{4}\mu_0 \frac{\mathscr{F}_{sr}^2}{g^2}\pi 2 r \ell g$$

$$W'_{fld} = \frac{\mu_0 \pi r \ell}{2g}\left( \mathscr{F}_f^2 + \mathscr{F}_a^2 + 2\,\mathscr{F}_f\,\mathscr{F}_a \cos(\theta) \right)$$

$$T_e = \frac{\partial W'_{fld}}{\partial \theta} = K\,\mathscr{F}_f\,\mathscr{F}_a \sin(\theta)$$

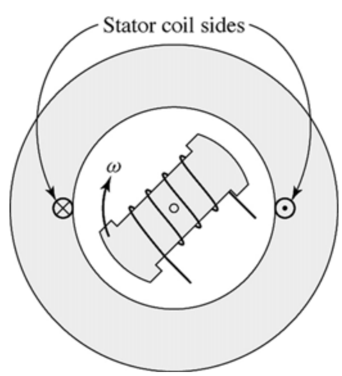
## EXAMPLES

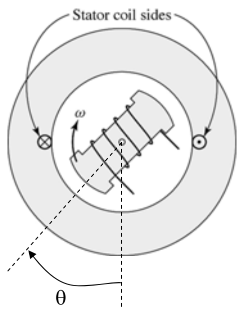
**Ex1: Figure on the right shows the two-pole revolving inside a smooth stator which carries a coil of 110 turns. The rotor produces a sinusoidal space distribution of flux at the stator surface; the peak value of the flux-density wave being 0.85T when the current in the rotor is 15A. The magnetic circuit is linear. The inside diameter of the stator is 11cm, and its axial length is 0.17m. The rotor is driven at a speed of 50 r/sec.**

**a. The rotor is excited by a current of 15A. Taking zero time as the instant when the axis of the rotor is vertical, find the expression for the instantaneous voltage generated in the open-circuited stator coil.**

**b. The rotor is now excited by a 50-Hz sinusoidal alternating current whose peak value is 15A. Consequently, the rotor current reverses every half revolution; it is timed to be at its maximum just as the axis of the rotor is vertical (i.e. just as it becomes aligned with that of the stator coil). Taking zero time as the instant when the axis of the rotor is vertical, find the expression for the instantaneous voltage generated in the open-circuited stator coil. This scheme is sometimes suggested as a dc generator without a commutator; the thought being that if alternative half cycles of the alternating voltage generated in part (a) are reversed by reversal of the polarity of the field (rotor) winding, then a pulsating direct voltage will be generated in the stator. Discuss whether or not this scheme will work.**

**Elementary generator for Problem 4.13. (Fitzgerald, 6th ed.)**






$$B_f = B_{peak}\sin(\theta)$$

Mean air gap flux per pole:

$$\phi_{avg/pole} = B_{avg}A_{per\ pole}$$

$$= \int_0^{\pi} B_{peak}\sin(\theta)dA$$

$$= \int_0^{\pi} B_{peak}\sin(\theta)\ell r d\theta$$

$$\phi_{avg/pole} = 2B_{peak}\ell r$$

$A_{per\ pole}$: surface spanned by a pole
One pole spans 180 ***elec. deg.***

**Problem 4.13**

part (a): The flux per pole is

$$\Phi = 2lRB_{\mathrm{ag1,peak}} = 0.0159 \quad \mathrm{Wb}$$

The electrical frequency of the generated voltage will be 50 Hz. The peak voltage will be

**Problem 4.13**

part (a): The flux per pole is

$$\Phi = 2lRB_{\mathrm{ag1,peak}} = 0.0159 \quad \mathrm{Wb}$$

The electrical frequency of the generated voltage will be 50 Hz. The peak voltage will be

$$V_{\mathrm{peak}} = \omega N\Phi = 388 \quad \mathrm{V}$$

Because the space-fundamental winding flux linkage is at is peak at time $t = 0$ and because the voltage is equal to the time derivative of the flux linkage, we can write

$$v(t) = \pm V_{\mathrm{peak}}\sin\omega t$$

where the sign of the voltage depends upon the polarities defined for the flux and the stator coil and $\omega = 120\pi$ rad/sec.

part (b): In this case, $\Phi$ will be of the form

$$\Phi(t) = \Phi_0\cos^2\omega t$$

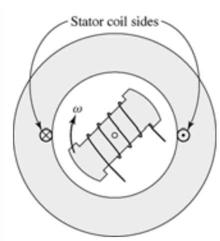
where $\Phi_0 = 0.0159$ Wb as found in part (a). The stator coil flux linkages will thus be

$$\lambda(t) = \pm N\Phi(t) = N\Phi_0\cos^2\omega t = \pm\frac{1}{2}N\Phi_0(1+\cos 2\omega t)$$

and the generated voltage will be

$$v(t) = \mp\omega\Phi_0\sin 2\omega t$$

This scheme will not work since the dc-component of the coil flux will produce no voltage.



**Ex2:** **Figure on the right shows a cross section of a machine having a rotor winding f and windings a and b whose axes are in quadrature. The self-inductance of each stator windind is $L_{aa}$ and of the rotor is $L_{ff}$. The air gap is uniform. The mutual inductance between a stator winding depends on the angular position of the rotor and may be assumed to be of the form**

$$M_{af} = M\cos\theta_0 \quad \text{and} \quad M_{bf} = M\sin\theta_0$$

**where M is the maximum value of the mutual inductance. The resistance of each stator winding is $R_a$.**

**a.** **Derive a general expression for the torque $T$ in terms of the angle $\theta_0$, the inductance parameters, and the instantaneous currents $i_a$, $i_b$ and $i_f$. Does this expression apply at standstill. When the rotor is revolving?**

**b.** **Suppose the rotor is stationary and constant direct currents $I_a = I_0$, $I_b = I_0$, $I_f = 2I_0$ are supplied to the windings in the directions indicated by the dots and crosses in the figure. If the rotor is allowed to move, will it rotate continuously or will it tend to come to rest? If the latter, at what value of $\theta_0$.**

**c.** **The rotor winding is now excited by a constant direct current $I_f$ while the stator windings carry balanced two-phase currents**

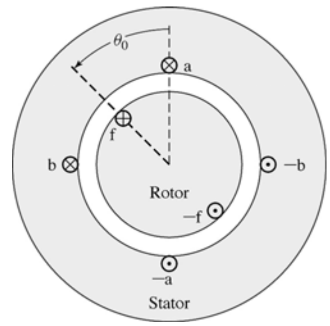
$$i_a = \sqrt{2}I_a\cos\omega t \quad \text{and} \quad i_b = \sqrt{2}I_a\sin\omega t\,.$$

**The rotor is revolving at synchronous speed so that its instantaneous angular position is given by $\theta_0 = \omega t - \delta$, where $\delta$ is a phase angle describing the position of the rotor at $t = 0$. The machine is an elementary two-phase synchronous machine. Derive an expression for the torque.**

**d.** **Under the conditions of part (c), derive an expression for the instantaneous terminal voltages of stator phases a and b.**

**Elementary cylindrical-rotor, two-phase synchronous machine for Problem 4.22**

**(Fitzgerald, 6th ed.)**




**Problem 4.22**

part (a):

$$\begin{aligned} T &= i_a i_f \frac{dM_{af}}{d\theta_0} + i_b i_f \frac{dM_{bf}}{d\theta_0} \\ &= M i_f\,(i_b\cos\theta_0 - i_a\sin\theta_0) \end{aligned}$$

This expression applies under all operating conditions.

part (b):

$$T = 2MI_0^2(\cos\theta_0 - \sin\theta_0) = 2\sqrt{2}\,MI_0^2\sin(\theta_0 - \pi/4)$$

Provided there are any losses at all, the rotor will come to rest at $\theta_0 = \pi/4$ for which $T = 0$ and $dt/d\theta_0 < 0$.

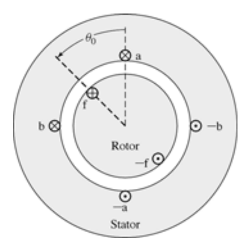
part (c):

$$\begin{aligned} T &= \sqrt{2}\,MI_aI_f(\sin\omega t\cos\theta_0 - \cos\omega t\sin\theta_0) \\ &= \sqrt{2}\,MI_aI_f\sin(\omega t - \theta_0) = \sqrt{2}\,MI_aI_f\sin\delta \end{aligned}$$

part (d):

$$\begin{aligned} v_a &= R_a i_a + \frac{d}{dt}\,(L_{aa}i_a + M_{af}i_f) \\ &= \sqrt{2}\,I_a(R_a\cos\omega t - \omega L_{aa}\sin\omega t) - \omega M I_f\sin(\omega t - \delta) \end{aligned}$$

$$\begin{aligned} v_b &= R_a i_b + \frac{d}{dt}\,(L_{aa}i_b + M_{bf}i_f) \\ &= \sqrt{2}\,I_a(R_a\sin\omega t + \omega L_{aa}\cos\omega t) + \omega M I_f\cos(\omega t - \delta) \end{aligned}$$



**Ex3:** **Figure on the right shows the cross section of a salient-pole synchronous machine having two identical stator windings a and b on a laminated steel core. The salient-pole rotor is made of steel and carries a field winding f connected to sp slip rings.**

**Because of the nonuniform air gap, the self- and mutual inductances are functions of the angular position $\theta_0$ of the rotor. Their variation with $\theta_0$ can be approximated as:**

$L_{aa} = L_0 + L_2 \cos 2\theta_0$, $L_{bb} = L_0 - L_2 \cos 2\theta_0$ and $M_{ab} = L_2 \sin 2\theta_0$

**where $L_0$ and $L_2$ are positive constants. The mutual inductance between the rotor and the stator windings are functions of $\theta_0$**

$M_{af} = M \cos \theta_0$ and $M_{bf} = M \sin \theta_0$

**where M is also a positive constant. The self inductance of the field winding, $L_{ff}$ is constant, independent of $\theta_0$.**

**Consider the operating condition in which the field winding is excited by direct current $I_f$ and stator windings are connected to a balanced two-phase voltage source of frequency $\omega$. With the rotor revolving at synchronous speed, its angular position will be given by $\theta_0 = \omega t$.**

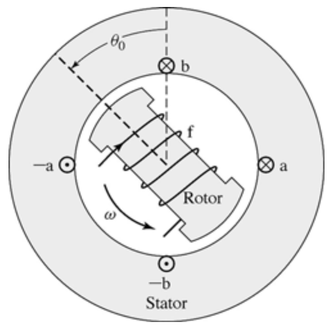
**Under this operating condition, the stator currents will be of the form**

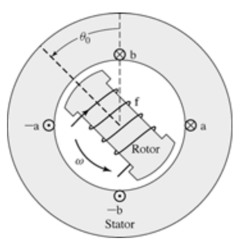
$$i_a = \sqrt{2} I_a \cos(\omega t + \delta) \text{ and } i_b = \sqrt{2} I_a \sin(\omega t + \delta).$$

a. **Derive an expression for the electromagnetic torque acting on the rotor.**
b. **Can the machine be operated as a motor and/or a generator?**
c. **Will the machine continue to run if the field current $I_f$ is reduced to zero?**

**Schematic two-phase, salient-pole synchronous machine for Problem 4.24.**

**(Fitzgerald, 6th ed.)**




**Problem 4.24**

part (a):

$$\begin{aligned} T &= \frac{i_a^2}{2}\frac{dL_{aa}}{d\theta_0} + \frac{i_b^2}{2}\frac{dL_{bb}}{d\theta_0} + i_a i_b \frac{dL_{ab}}{d\theta_0} + i_a i_f \frac{dM_{af}}{d\theta_0} + i_b i_f \frac{dM_{bf}}{d\theta_0} \\ &= \sqrt{2}\, I_a I_f M \sin\delta + 2I_a^2 L_2 \sin 2\delta \end{aligned}$$

part (b): Motor if $T > 0$, $\delta > 0$. Generator if $T < 0$, $\delta < 0$.

part (c): For $I_f = 0$, there will still be a reluctance torque $T = 2I_a^2 L_2 \sin 2\delta$ and the machine can still operate.